Problem Set 8

1. Given a graph G, the maximum clique problem asks for the largest value of k such there is a set S consisting of k vertices, such that each pair of vertices in S is joined by an edge in G (such a set is called a k-clique). Assuming that $P \neq NP$ show that there is no polynomial time algorithm to solve the maximum clique problem. This problem is not in the right form to be NP-complete. What small change could you make to get a similar problem that's NP-complete?

2. You're given a set S of points in the plane. Your goal is to decide if there's a collection of m horizontal lines and n vertical lines that together cover all of S. Show that this problem is NP-complete.

3. A vector is s-sparse if it has at most s nonzero entries. The following problem arises in *compressed sensing*:

   Given a matrix $A \in \mathbb{R}^{n \times m}$, a vector $b \in \mathbb{R}^n$ and a number s, decide if there is an s-sparse vector $x \in \mathbb{R}^m$ such that $Ax = b$.

   Show that this problem is NP-complete. (Hint: Look at the 3-dimensional matching problem in the textbook. You can assume that 3D matching is NP-complete. Also note that Ax is a linear combination of s columns of A if x is s-sparse.)

4. Make up a problem involving any topic from the course and post it as its own post on Campuswire (tag it with "final exam"). Feel free to be creative. Respond to someone else's post with a solution or an otherwise valuable comment. The goal is for you to collectively make a study guide for the final.