Problem Set 5

In the following problems involving graphs, m always represents the number of edges, and n always represents the number of nodes. You should also justify the correctness of your algorithms wherever it's not obvious.

1. Prices are still rising in Inflationville. This time, the city has a collection of hubs that it would like to connect with a spanning tree. There are already some roads between the hubs which are not yet owned by the city. Each road e currently costs $c(e)$ dollars, but the costs of the roads are rising by 5 percent every day, and the city needs to spend a day to purchase a given road (only one road can be purchased at a time). Give an $O(m \log n)$ algorithm to decide which roads the city should purchase, and in what order to minimize cost. Give a careful proof that your algorithm is correct.

2. We have series of nodes which make up a network. Some pairs can be connected via a connection with bandwidth $b(e)$. We'd like establish a path through the network to send information between nodes s and t. We'd also like to maximize the bandwidth between s and t, where the bandwidth along a path is given by the minimum bandwidth of any of its edges.

   (a) Give an $O(m \log n)$ algorithm to find a path between s and t that maximizes the bandwidth.

   (b) On second thought, it would really make sense to establish a set of connections so that *all* pairs of nodes are connected by a path of maximum bandwidth. Give an efficient algorithm to find the smallest set of connections that can be established.

3. The goal of the problem is to implement a stable marriage algorithm. See the notebook.