Reconstructing piezoelectric responses over a lattice: adaptive sampling of low dimensional time series representations based on relative isolation and gradient size

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Abstract

We consider a $d-$dimensional lattice of points where, at each lattice point, a time series can be measured. We are interested in accurately reconstructing the full dataset using as few measurements as possible. Through a convex weighting of a point's relative isolation and relative gradient size, we assign a sampling priority. Our method, Relative Isolation and Gradient Size (RIGS Sampling) adaptively samples the time series and then reconstructs the time series over the entire lattice. We can capture the important dynamics and achieve a relative ℓ_2 reconstruction error of less than 10% with a 47% reduction in measurements, and less than 5% with a 25% reduction in measurements.

Problem Formulation

We are provided measurements of the piezoelectric responses of a ferroelectric material at various points and voltages. The points and voltages form a lattice of the form (x_i, y_j, V_k) where (x_i, y_j) is a position and V_k is a voltage. The piezoelectric response $p(t; x, j, V)$ is a time series sampled up to $t = T$ at a point (x, y) and voltage V. **We seek a method that, for a fixed fraction of lattice points sampled, accurately reconstructs the time series at all points of the lattice.**

Method (RIGS Sampling)

Hyperparameters: choose $0 \leq w \leq 1$ to be the gradient weight, f to be the fraction of lattice points sampled, R to be the reconstruction frequency, and k to be the number of nearest neighbors used in reconstruction.

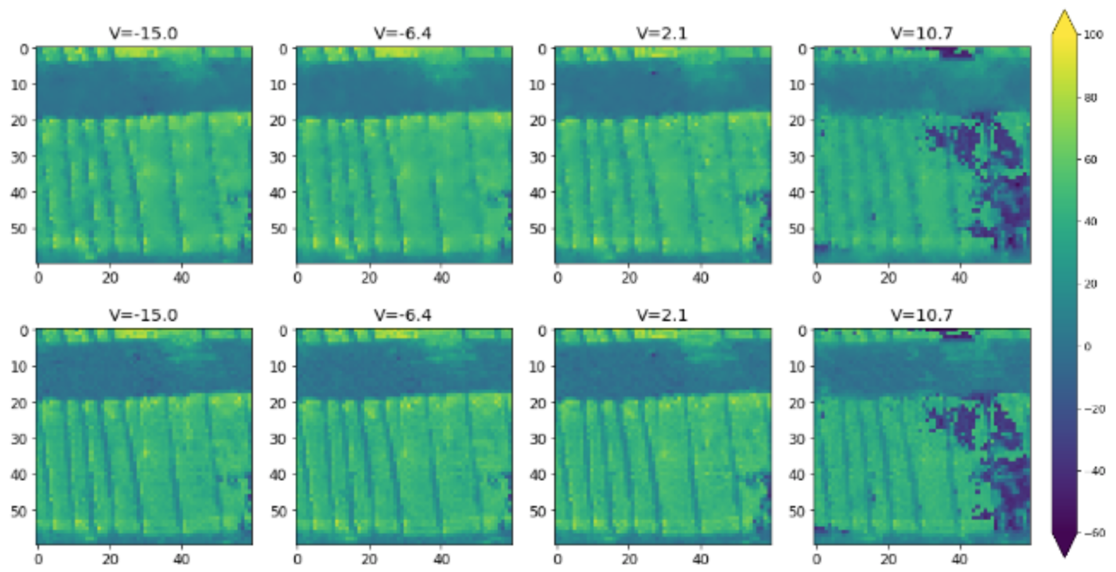
A first sampling point of the lattice is selected. Then, until sampling is complete:

- Each unsampled lattice point u is assigned a gradient score G_u by (1) estimating $|\nabla \bar{p}|$ at all lattice points and then (2) normalizing in ℓ_2. Here $\bar{p}$ is the time-average value value of p at a lattice point.
- Each unsampled lattice point u is assigned an isolation score I_u by (1) computing a density proxy of form $\sum_{s \in S} e^{-|s-u|^2}$ where S is a set of points based on those sampled, (2) inverting this density, and (3) normalizing in ℓ_2.
- Points with the largest sampling priority $S_u = wG_u + (1-w)I_u$ are sampled.
- Every R iterations, reconstruct $\bar{p}$ over the full lattice using the k nearest neighbors.

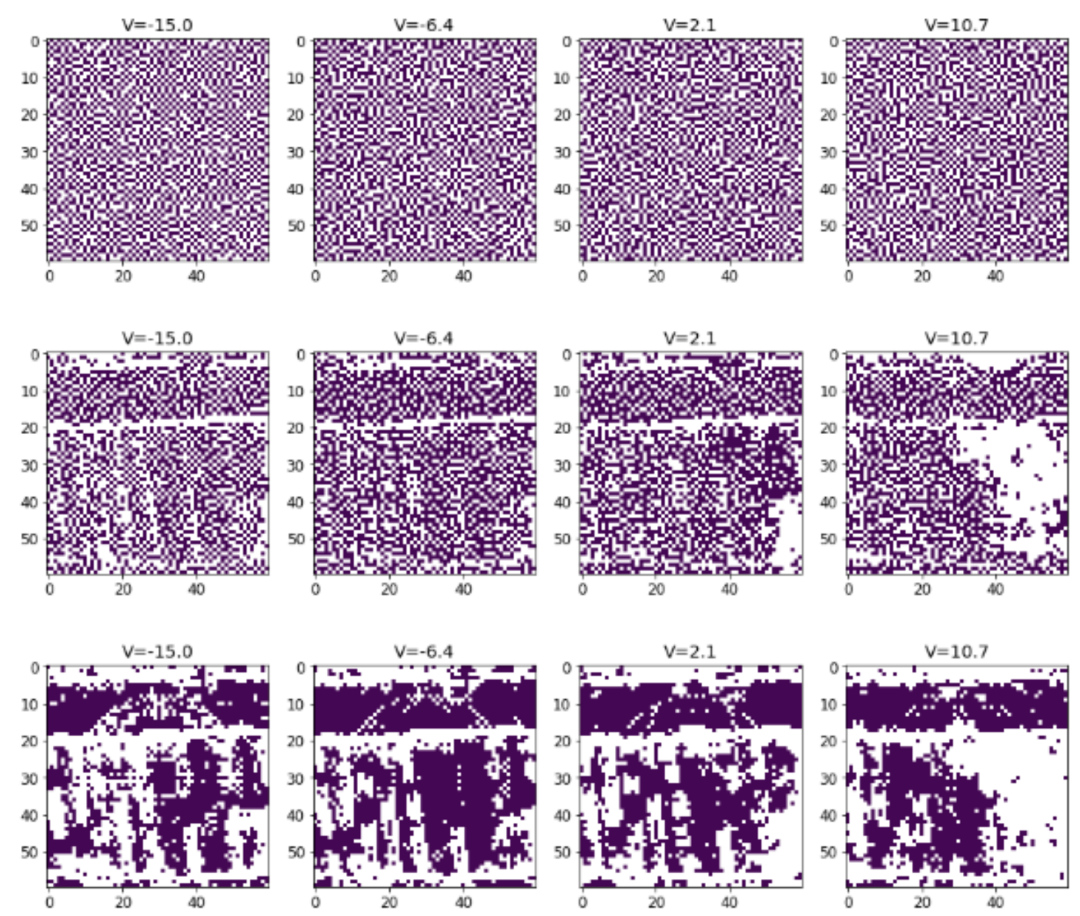
After sampling is done, $p_{ijk}(t)$ is estimated through an k nearest neighbor regression in each time slice.

Results

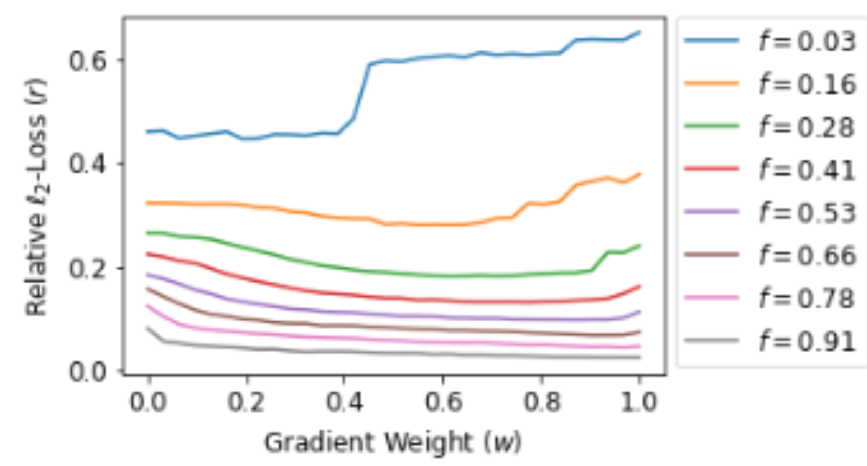
Visual performance. The top row in the figure below shows our reconstruction at four voltage slices, with **40%** sampling, compared to the bottom row which shows the ground truth. This illustrates that our method recovers sharp edges quite well – see the bottom right-hand corner in the figure above.



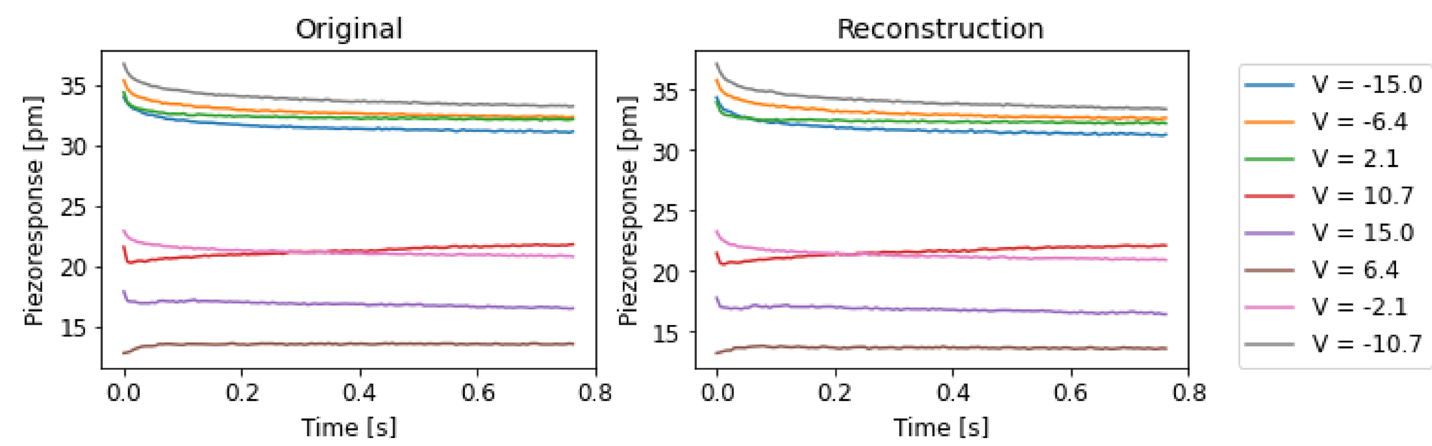
Effects of adaptivity. To illustrate the effects of adaptivity, we show the masks generated when the gradient weight is set to 0, 0.5 and 1 respectively (0 corresponds to non-adaptive sampling). A higher gradient weight results in the samples being more concentrated near edges.



Quantitative accuracy. In the following figure, we show the ℓ_2 loss of our reconstruction as a function of the gradient weight w, and the fraction of sampled points f. The reconstruction accuracy generally improves up to $w = 0.8$. However setting w too large hurts; it is still important to focus on sampling isolated points which are far from the points sampled so far.



Time series reconstruction. The figure below shows the average piezoelectric response as a function of time for various voltages with 50% of points sampled. Our method recovers the average behavior nearly perfectly.



Acknowledgments

WJS and DN acknowledge funding from NSF BIGDATA #1740325 and NSF DMS #2011140.